Math 1B Practice Midterm 2 Answers, July 27 2011

- 1. (a) The series $\sum_{n=1}^{\infty} \frac{1}{n^{1.2}}$; C (*p*-test);
 - (b) The sequence $\frac{n}{n-\ln n}$; C (divide top and bottom by n, or use L'Hôpital);
 - (c) The series $\sum_{n=1}^{\infty} \frac{n^n}{n!}$; D (divergence test);
 - (d) The sequence $(\sin 2n)^n/n$; C ($|\sin 2n|$ bounded by 1);
 - (e) The series $\sum_{n=1}^{\infty} \frac{2^n n^3}{5^n n^{10}}$; C (looks geometric with r = 2/5).
- 2. The moment of a piece is the mass times x-distance. The mass is given to us as $f(x_i)\delta x\rho$, so the moment of a piece is $x_i f(x_i)\delta x\rho$, and so the total moment is $\sum_{i=1}^n x_i f(x_i)\rho\delta x$. The formula for the x-coordinate of the center of mass is

$$\frac{1}{A} \int_{a}^{b} x f(x) dx.$$

3. The r chosen lies between L and 1. Thus, if L < 1 then r < 1 and if L > 1, then r > 1. A geometric series converges if and only if |r| < 1. The geometric series bounds a final piece of $\sum_{n=1}^{\infty} a_n$ from below when 1 < r < L and from above when 1 > r > L, and so the Comparison Theorem then gives us that if L < 1 we have convergence and if L > 1 we have divergence.