Math 1B Midterm 1, July 8 2011, 3:00pm-4:00pm

Justify all answers and show all work. Clearly mark where your answer is written in your blue book. Problems 2 and 5 are worth 15 points. All other problems are worth 14 points.

1. Evaluate the integral $\int_1^2 x^2 \cdot x^2 \ln(x^2) dx$.

2. Evaluate the integral $\int_1^{\sqrt{2}} \frac{\sqrt{2-x^2}}{x^2} dx$.

3. Use Simpson's rule with three points to estimate the integral $\int_0^2 \frac{dx}{3^x+1}$. As a function, the fourth derivative of $\frac{1}{3^x+1}$ is always less (in absolute value) than $\frac{1}{3^x+1}$ on the interval $[0, 2]$. Based on this fact and your estimate of the integral, could the actual value of the integral be $\frac{17}{30}$?

4. Evaluate the integral $\int \frac{4x}{x^2-2x-3} dx$.

5. For what values of p does the integral $\int_1^\infty \frac{dx}{x^p}$ converge? For such values of p, what does the integral evaluate to?

6. State the formula for the arc length of a curve given by $x = g(y)$, for $c \leq y \leq d$. Use it to find the arc length of the curve given by $x = \ln(\cos(y))$, for $-\pi/4 \leq y \leq \pi/4$.

7. We want to approximate the area of a surface coming from the revolution of a differentiable curve $y = f(x)$ around the x-axis, as x ranges over an interval $[a, b]$. First, we divide the interval $[a, b]$ into n equal-length subintervals by picking $n + 1$ equally-spaced points, $x_0 = a, x_1, \ldots, x_{n-1}, x_n = b$. This gives us $n + 1$ points on the curve, $P_0 = (a, f(a)), P_1 = (x_1, f(x_1)), \ldots, P_{n-1} = (x_{n-1}, f(x_{n-1})), P_n = (b, f(b))$. Write down the approximation to surface area that comes from connecting the P_i points by straight line segments, and then revolving these line segments around the x-axis. You should use the fact that if we take a straight line segment of length l, with the left endpoint having y-coordinate r_1 and right endpoint having y-coordinate r_2, then revolving it around the x-axis yields a surface with area $\pi(r_1 + r_2)l$.

Facts about trigonometric functions:

- $\sin^2(\theta) + \cos^2(\theta) = 1$;
- $\tan^2(\theta) + 1 = \sec^2(\theta)$;
- $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$;
- $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$;
- $\sin(\pi/4) = \cos(\pi/4)$;
- $\frac{d}{d\theta}\cot(\theta) = -\csc^2(\theta)$;
- $\int \sec(\theta) d\theta = \ln(|\sec(\theta) + \tan(\theta)|) + C$;